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Progetto di Ricerca e Piano di Attività

Well-posedness of degenerate McKean-Vlasov stochastic differential equations

The objective of this project is to advance the field of Stochastic Differential Equations (SDEs) research. Specifically, the focus lies in investigating weak and strong well-posedness for systems of degenerate SDEs of McKean-Vlasov type, characterized by rough coefficients.

1 State of art

The problem of well-posedness for an SDE consists in determining the existence and uniqueness of a solution to the equation. These two properties can be studied in their weak or strong formulation. Roughly speaking, weak existence and uniqueness concern the law of a solution, while strong existence and uniqueness are trajectory-wise properties (that hold almost surely).

In recent decades, numerous authors have extensively investigated the well-posedness of degenerate SDEs. Specifically, this project focuses on examining fully-degenerate equations, where diffusion influences only a subset of the spatial variables. The seminal work by Kolmogorov [7] introduced a particular example of a wide class of degenerate equations that is still largely studied. These kind of SDEs naturally arise, for example, in the study of physical kinetic models as well as in heat conduction models or in financial modeling of Asian options (see for example [5], [1], [11]).

The well-posedness of an SDE is strongly related with the regularity of its coefficients. When the coefficients of a non-degenerate equation are Lipschitz continuous, the strong well-posedness is a classical result, but it may fail when the assumptions are weaker. First results in this direction can be found in [13] and [14]. Concerning degenerate models, in recent years there is a vast literature on well-posedness for SDEs with Hölder continuous drifts (see for example [2] and [4]).

This project focuses on the study well-posedness for SDE of McKean-Vlasov type (MKV). These equations arise as limit of large interacting particle systems and are characterized by some kind of dependence of the coefficients on the law of the solution (see for example [12]). For a non-degenerate MKV SDE, weak and strong well-posedness are proved in [9] under relaxed regularity condition on the coefficients. In [3] strong uniqueness for a non-degenerate MKV SDE is established when coefficients satisfy a Hölder condition, also in the law argument with respect to the Wasserstein metric. In [6], the authors prove weak uniqueness for a kinetic MKV equation with singular drift in a Besov space.

2 Project Overview and Activity Plan

This project focuses on the study of weak and strong well-posedness for degenerate MKV SDE with coefficients that are Hölder continuous in space and measurable in time. One potential approach involves examining the solution of the backward Kolmogorov equation linked to the SDE. Establishing the existence of a solution at the PDE level could lead to proving the strong uniqueness of the SDE solution, as demonstrated in, for instance, [14]. This, in conjunction with weak existence, implies strong existence under mild assumptions on the coefficients, as outlined in the classical Yamada and Watanabe theorem [13].

Another viable approach involves employing a parametrix technique to explore the existence and regularity of a solution. This technique has been recently utilized in [10] and [8] to establish optimal regularity for the fundamental solution of a backward Kolmogorov equation (which represents the transition density of the solution of the associated SDE) and its associated Cauchy problem. Building upon the insights from [9] and [3], weak and strong uniqueness for kinetic MKV SDEs with Hölder continuous coefficients can be studied, respectively.

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